Strength of Mechanical Constructions

Introduction

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Organisational information

Consultations

- Tuesday and Wednesday, 09:30-10:15
- room: MC404
- building: A5 (Piotrowo campus)

Subject organisation

- lecture: 15 hours
- exercise: 15 hours

Credit

- lecture + exercise: final project covering knowledge from the lectures

Goal of the subject

The goal of the subject is

to understand how machine elements and materials work under external load

What knowledge do we need?

- mechanics of materials
  - to understand deformation
- material science
  - to be familiar with the structure of the material
- experiment
  - to obtain the mechanical properties and understand the mechanics of deformation and destruction
- mathematics
  - to describe the relation between quantities

Scope of the subject

- behaviour of material under load
- introduction of basic concepts: displacement, deformation, strain, stress
- energy of elastic deformation
- application of energy methods in mechanics
  - Castigliano's theorem
  - stability of structures
  - impact load
- curved bars
- thick cylinders
- introduction to shell structures

Bibliography

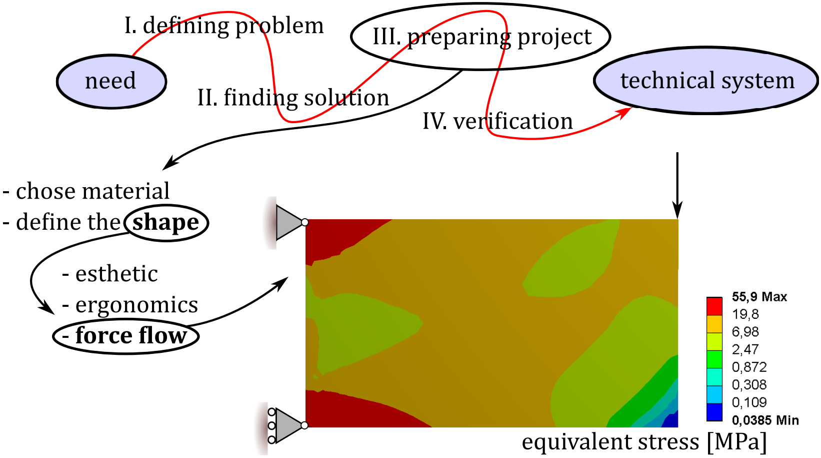
1. Hibbeler RC. *Statics and Mechanics of Materials (5th ed.)*, Pearson, Boston, 2016
2. Gere JM., Goodno BJ. *Mechanics of Materials*, Cengage Learning, Australia, 2009
3. Timoshenko S. *Strength of Materials, part II: Advanced Theory and Problems*, D. Van Nostrand Company, Inc., New York, 1947
4. Nash WA. *Schaum's Outline of Theory and Problems of Strength of Materials*, McGraw-Hill, New York, 1998.
5. Hartog D. *Advanced Strength of Materials*, Dover Publications, Inc., New York, 1987
6. Boresi AP., Schmidt RJ. *Advanced Mechanics of Materials*, Joh Willey & Sons, Inc., New York, 2003.

Scope of presentation

1. Introduction
   - Mechanics of materials in design process
2. Behaviour of material under load
   - Types of deformations
   - Modes of failures
3. Basic definitions
   - Definition of stress; internal forces
   - Deformation; displacement and strains
   - Physical relations

Contents

Shaping structural elements



Shaping structural elements
Requirements

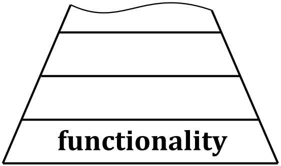
Functionality

ability of a structure to perform its function

To perform its functions the structure may not:

- loose its integrity – *strength condition*
- undergo excessive deformation – *stiffness condition*
- loose its stability – *stability condition*

$$\left.\begin{array}{l}\sigma \leq \sigma_{allow}\\ \delta \leq \delta_{allow}\\ \boldsymbol{F} \leq \boldsymbol{F}_{cr}\end{array}\right\} = \text{functionality}$$



Shaping structural elements

Requirements

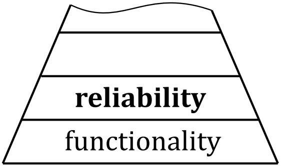
Reliability

functionality maintained for assumed amount of time

To add the time factor one has to understand the processes occurring in the structure and in the material during the operational time. These are:

- load and support conditions
- environment impact
- surface processes

$$\left.\begin{array}{l} \sigma \leq \sigma_{allow} \\ \delta \leq \delta_{allow} \\ \boldsymbol{F} \leq \boldsymbol{F}_{cr} \end{array}\right\} + \text{time} = \text{reliability}$$



Remember about the weakest link!

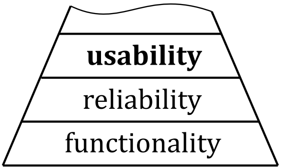
Shaping structural elements

Requirements

Usability

ease of use of the product; forgiving design

$$\left.\begin{array}{l}\sigma \leq \sigma_{allow} \\ \delta \leq \delta_{allow} \\ \boldsymbol{F} \leq \boldsymbol{F}_{cr}\end{array}\right\} + \text{time} + \text{ergonomics} = \text{usability}$$



Shaping structural elements

Requirements

To fulfil the functionality requirement the designer must

- predict the possible mode of failure,
- chose failure criteria.

To predict the mode of failure

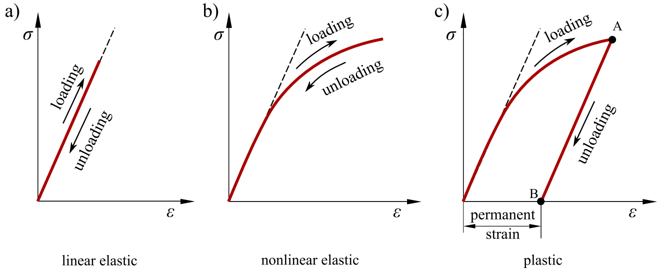
- stress analysis is required,
- behaviour of material under load should be analysed,
- history of loading should be assumed.

Contents

Behaviour of material under load

Types of materials

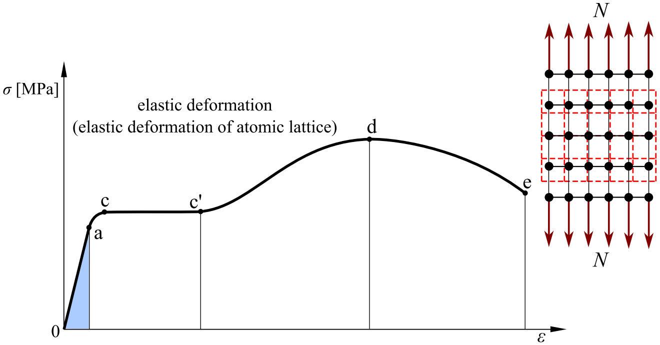
- regular operation of the structure usually require the material to stay in elastic range
- to predict the failure of the material plastic behaviour should be also considered



Behaviour of material under load

Stages of deformation

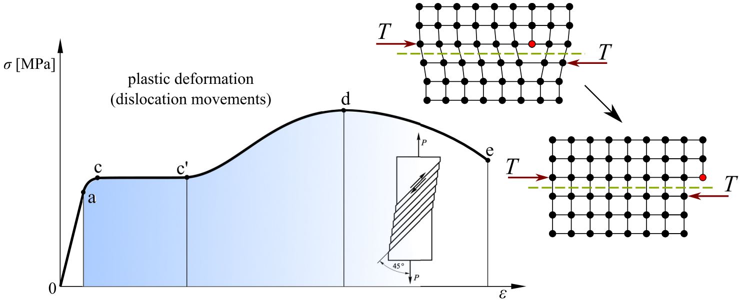
- in elastic range atomic lattice deforms under the load and return to its initial shape after reloading



Behaviour of material under load

Stages of deformation

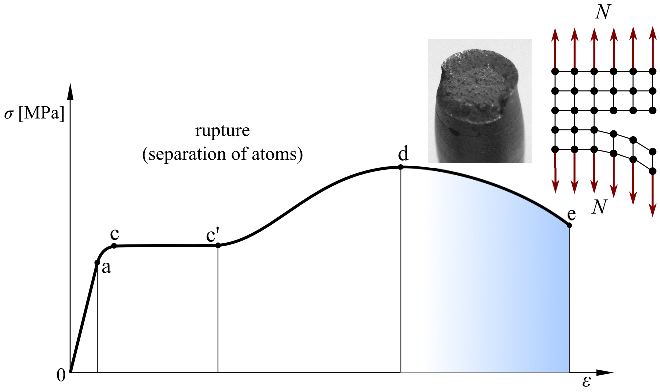
- in plastic range the dislocations start to move and the deformation is permanent



Behaviour of material under load

Stages of deformation

- if the internal forces exceed the atomic forces the material fails due to rupture



Contents

Modes of failures

Why does a structure fail?

- improper design
  - stress, material, shape
- manufacturing defects
  - porosity, cracks, poor surface finish
- wrong operation and maintenance
  - lubrication, vibrations, wrong replacement
- environmental effects
  - corrosion, temperature

Modes of failures

Examples of improper design:

- underestimated services stress
- undesirable geometry
  - stress concentrators
  - inadequate radii at corners
  - inaccessibility for inspection
  - difficult to fabricate
- improper choice of materials
- improper choice of heat treatment
- environmental effects
- ad hoc modifications

Modes of failures

Failure by excessive deflection

- deflection in elastic range (static equilibrium)
  - define maximum elastic deformation
- amplitudes of vibration
  - define mode of vibration and natural frequencies

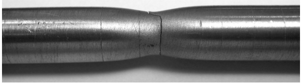
Design hint

the most effective method to decrease maximum deformation or amplitude of vibrations is to modify the shape of the structure, *e.g.* the cross-section of a structural member

Modes of failures

Failure by general yielding

- yield point has been exceeded
  - stress concentration are not taken into account
  - apply material with higher yield strength

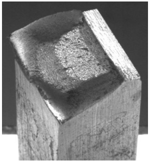


- creep phenomenon appeared
  - creep – time-dependent inelastic strain under sustained load and elevated temperature
  - consider: possibility of creep, creep failure, creep fracture

Modes of failures

Failure by fracture

- sudden fracture of brittle material
  - failure criteria: ultimate strength
- fracture of flawed member
  - failure criteria: notch toughness
- progressive fracture (fatigue)
  - failure criteria: fatigue strength



Modes of failures

Failure by instability

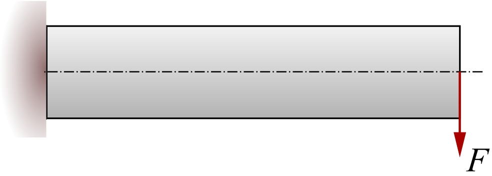
- sudden (catastrophic) change of the shape of a structure
  - define the critical load

Contents

What are we looking for?

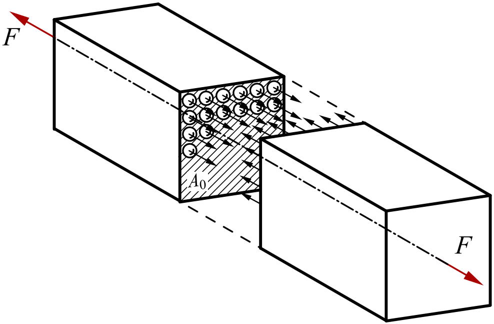
To analyse structure one has to find the relations between parameters in a micro and macro scale

- strains-displacements
- stresses-forces
- stress-strain



Definition of stress

- elastic body subjected to external load undergoes deformation
- it doesn't lose its integrity due to internal forces
- these forces are molecular forces acting between atoms



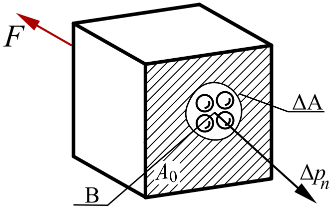
Definition of stress

- let's introduce the internal force Δp_n which is the sum of all forces spread on the area ΔA
- the measure of intensity of internal forces, **average stress**, is defined as

$$p_{n,av} = \frac{\Delta p_n}{\Delta A}$$

- if the area ΔA goes to 0 we obtain a **stress at point**

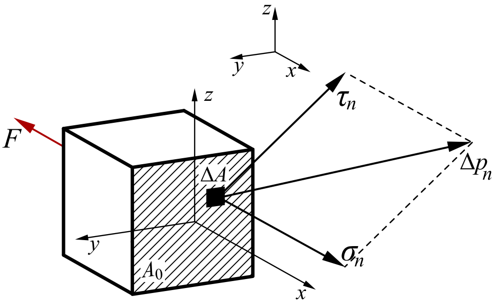
$$p_n = \lim_{\Delta A \to 0} \frac{\Delta p_n}{\Delta A}$$



Definition of stress

In general case the vector of stress can be directed in any direction and it can be decomposed into two components:

- normal to the cross-section → normal stress σ_n
- tangent to the cross-section → shear stress τ_n



Definition of stress

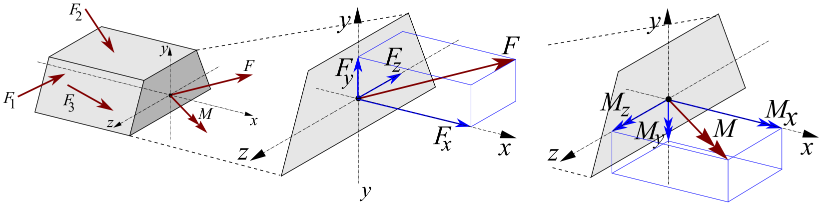
- stress, as well as the components of stress, can be defined at any point of any cross-section
- let's consider stress at point A of the plane yz of the Cartesian coordinates system; the axis x is normal to this plane.
- the stress vector p_n can be decompose on three components:
  - normal stress: σ_x
  - shear stress: τ_{xy}
  - shear stress: τ_{xz}

Components of stress

- in a similar way the stress can be described on two other planes
- consequently, one obtains 9 stress components describing the stress state at a point; all of them are functions of three coordinates
- indexes at particular components indicate:
    - direction normal to the analysed plane – first index
    - direction of action of the component – second index

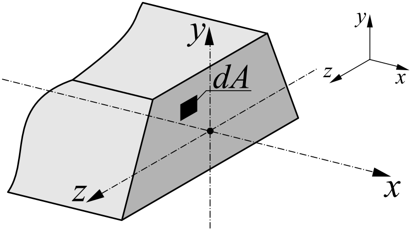
Internal forces in terms of stress

- from mechanics it is known that any system of forces acting on a body can be reduced to one force F and one moment M
- if we say about internal forces acting on the cross-section theirs vectors are fixed at the centroid C and can be decomposed into 6 components having some specific meaning



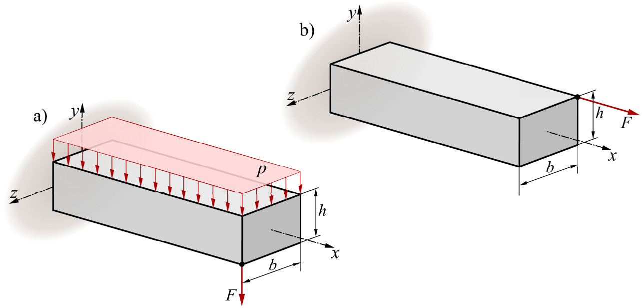
Internal forces in terms of stress

All internal forces can be defined in terms of stress components



Basic concepts
Free body diagram

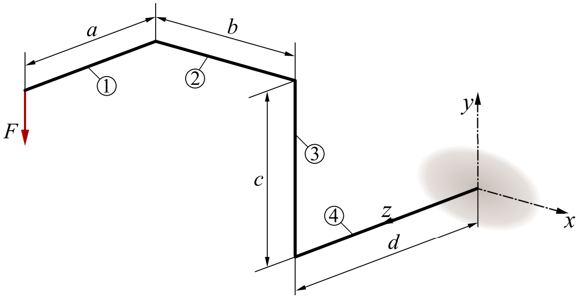
Before starting calculations there is a need to transfer the load applied to the real structure to the model of this structure. A free body diagram must be prepared.



Basic concepts

Method of section

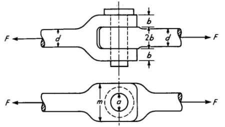
The relation between the applied load, which is known, and the internal forces can be determined with the use of method of section.



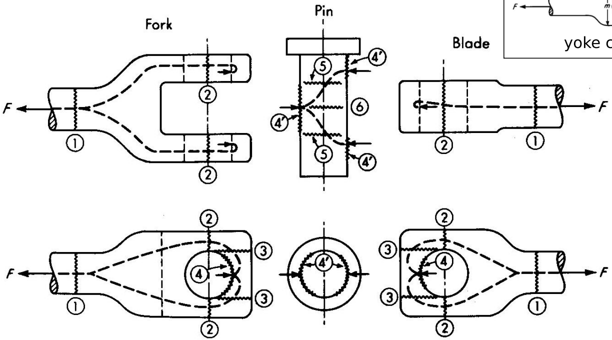
Basic concepts

Force flow

The critical areas of stress in a structure can be easily determined with the use of force flow concept.



yoke connection



Contents

Deformation of elastic body

Deformation

change of shape or size or both of the body

During deformation relative position of points of the body change.

If position of the body changes but relative position of its points do not change one has to do with rigid body motion.

Deformation of the body is possible only when relative position of its points changes.

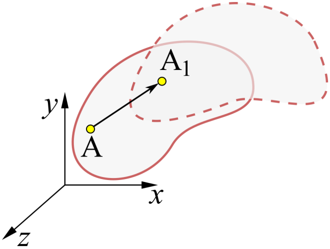
Displacement of the point of the body

Displacement

change of position of the point of the body

In Cartesian coordinate system there are three component of displacement which are functions of coordinates:

- $u = u(x, y, z)$
- $v = v(x, y, z)$
- $w = w(x, y, z)$

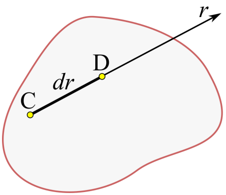


Displacement is positive if its direction coincide with the positive direction of the axis.

Deformation of the body at point

Linear deformation

Let's consider the linear deformation of a body at point 'C'. To do this we take the linear element 'CD' directed along the r direction.



Deformation of the body at point

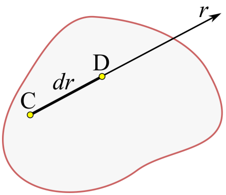
Linear deformation

Let's consider the linear deformation of a body at point 'C'. To do this we take the linear element 'CD' directed along the r direction.

Relative linear deformation – normal strain

$$\varepsilon_r = \lim_{CD \to 0} \frac{\Delta CD}{CD}$$

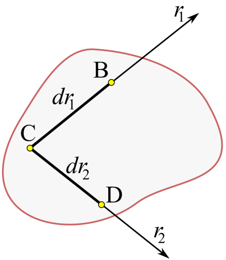
ε_r can be different in different directions passing through point 'C'.



Deformation of the body at point

Angular deformation

Let's consider the angular deformation of a body at point 'C'. To do this we take two linear elements 'CB' and 'CD' directed along the r_1 and r_2 directions, respectively.



Deformation of the body at point

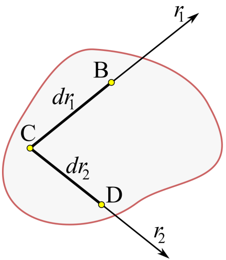
Angular deformation

Let's consider the angular deformation of a body at point 'C'. To do this we take two linear elements 'CB' and 'CD' directed along the r_1 and r_2 directions, respectively.

Angular deformation – shearing strain

$$\gamma = \alpha_1 + \alpha_2$$

γ can be different for different pairs of linear elements passing through point 'C'.



Deformation of the body

In Cartesian coordinate system there are **three** components of displacement and **six** components of deformation all of which are functions of coordinates (x, y, z):

- $u(x, y, z)$
- $v(x, y, z)$
- $w(x, y, z)$

- $\varepsilon_x(x, y, z)$
- $\varepsilon_y(x, y, z)$
- $\varepsilon_z(x, y, z)$
- $\gamma_{xy}(x, y, z)$
- $\gamma_{yz}(x, y, z)$
- $\gamma_{zx}(x, y, z)$

Knowing the functions of displacements, u, v, w, one can describe the deformation of the whole body – each point of the structure.

Deformation of the body

Functions of displacement and functions of deformation describe the same phenomenon but in a different way. Thus there must be some relation between them.

$$u(x,y,z) \;\; ? \;\; \varepsilon(x,y,z)$$

To find this relation we apply a geometric (kinematic) approach, which means that the stress as well as physical properties of material are not considered.

This way the obtained formulae will be valid for any material and any state of stress.

Deformation of the body

The assumptions which will help to formulate strain-displacement relations:

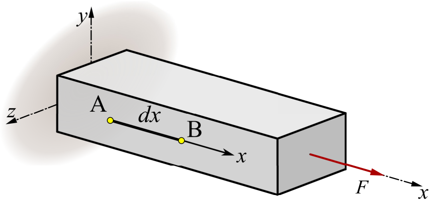
- the body is continuous and homogeneous,
- the body is stiff enough to assure the linear relation between displacements and deformations.

High stiffness means that the displacements are small when compare to the dimensions of the body.

Deformation in one dimensional problems

Axially loaded members

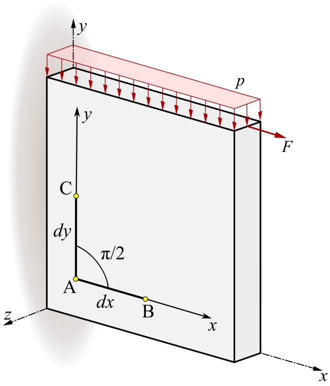
Find relation between strains and displacements for one dimensional state of stress.



Deformation in two dimensional problems

Plane state of stress

Find relation between strains and displacements for two dimensional state of stress.



Deformation of the body

For three dimensional problem there are six relations between strains and displacements:

NORMAL STRAIN

- $\varepsilon_x = \frac{\partial u}{\partial x}$
- $\varepsilon_y = \frac{\partial v}{\partial y}$
- $\varepsilon_z = \frac{\partial w}{\partial z}$

SHEAR STRAIN

- $\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$
- $\gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}$
- $\gamma_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$

Contents

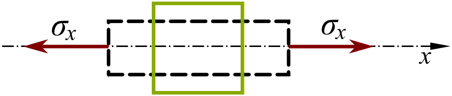
Physical relations – Hooke's law

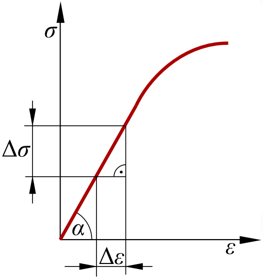
One dimensional problem

In one dimensional problems the Hooke's law can be determined from the static tensile test.

We neglect the deformation of the body in a transverse direction assuming that only the length of the body increases.

$$\sigma_x = E\varepsilon_x$$





Physical relations – Hooke's law

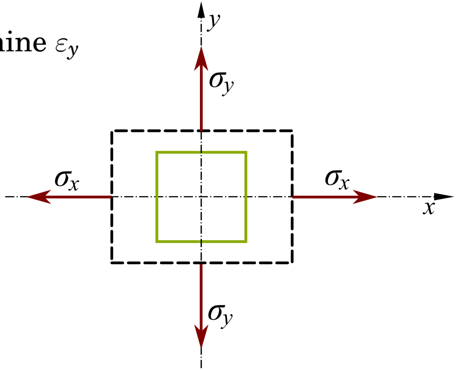
Two dimensional problem

In plane state of stress deformation appear in two directions. These deformations are related with the Poisson's ratio

$$\nu = -\frac{\varepsilon_y}{\varepsilon_x}$$

Knowing ε_x one can determine ε_y assuming that ν is given

$$\varepsilon_y = -\nu\varepsilon_x$$



Physical relations – Hooke's law

Two dimensional problem

In plane state of stress deformation appear in two directions. These deformations are related with the Poisson's ratio

$$\nu = -\frac{\varepsilon_y}{\varepsilon_x}$$

Knowing ε_x one can determine ε_y assuming that ν is given

$$\varepsilon_y = -\nu\varepsilon_x$$

$$\begin{cases} \sigma_x = \dfrac{E}{1-\nu^2}\left(\varepsilon_x + \nu\varepsilon_y\right) \\ \sigma_y = \dfrac{E}{1-\nu^2}\left(\varepsilon_y + \nu\varepsilon_x\right) \end{cases}$$