Strength of Mechanical Constructions

Analysis of shell structures

Paweł JASION, PhD. Eng.
e-mail: pawel.jasion@put.poznan.pl
www:pawel.jasion.pracownik.put.poznan.pl

Poznan University of Technology
Institute of Applied Mechanics
Division of Strength of Materials and Structures

Table of content

Contents

Application of shells

- variety of shapes
- vicious behaviour
- various applications

Application of shells

Broad application of shells comes from their advantages which are:

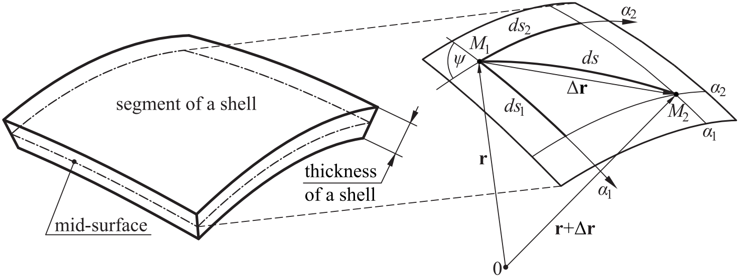
- effective load carrying
- high strenght-to-mass ratio
- high stiffness
- covering large spaces
- aesthetics

Contents

Definition of shell

Shell

structural element the thickness of which is much smaller than the other two dimensions



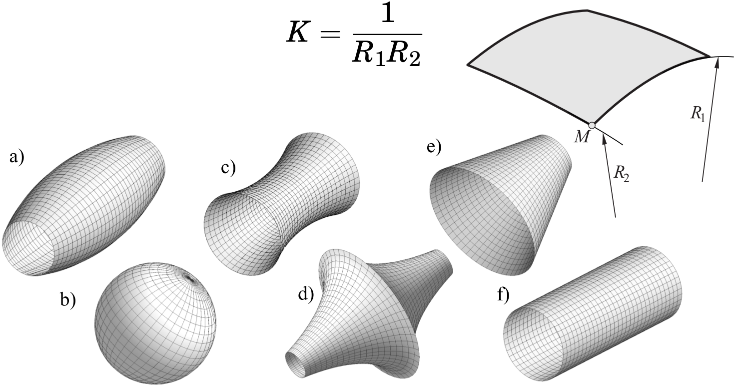
Definition of shell

- depending on the thickness one can defined:
    - thin shells, for which $(t/R)_{max} < 1/20$
    - thick shells, for which $(t/R)_{max} > 1/20$
- in the above t is the thickness of the shell and R is the radius of curvature

Shells of revolution

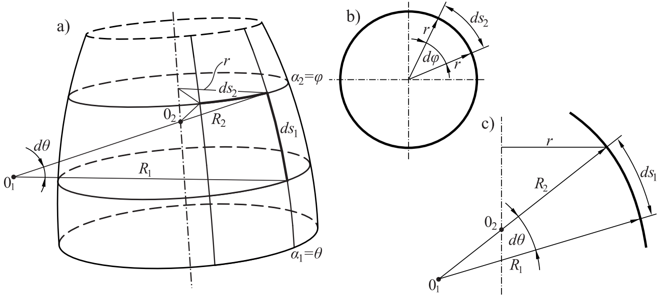
- the most popular shape of shell is a shell of revolution
- depending on the geometry one can distinguish shells with positive, negative or zero Gaussian curvature, defined as

$$K = \frac{1}{R_1 R_2}$$



Definition of shell

- geometry of shell of revolution is determined by two main radii of curvature: meridional R_1 and circumferential R_2



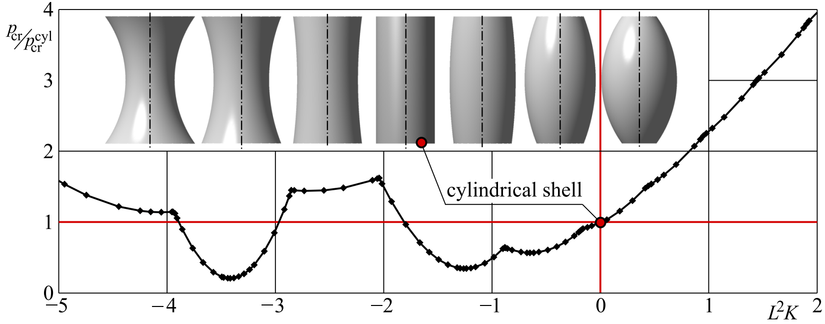
- differential relation: $\dfrac{d(R_2 \sin\theta)}{d\theta} = R_1 \cos\theta$

Load carrying mechanism

- the shell transfers the transverse load mainly through tensile and compressive stresses – membrane stress
- they are equally distributed through the thickness of the shell; such state of sterss is called the membrane state of stress
- the advantage of membrane state is that
    - it allows to take full advantages of the mechanical properties of the material – in pure tension, all fibers on the cross section are equally strained
    - for a given value of load the membrane stress are always smaller than the bending ones

Load carrying mechanism

- the effectiveness of the shell as a structural element is related with they curvature and thin-walld
- thanks to curvature, the shell gains spatial rigidity, and the load is distributed to membrane forces



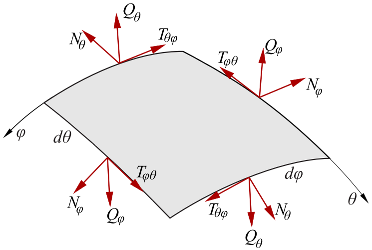
Load carrying mechanism

- thanks to thin-walled character, the shell gains lightness, and locally occurring bending stresses quickly disappear
- the disadvantage of thin walles is that in the case of compressive membrane stresses, the shell may buckle

Internal forces in shell

The main load carrying by plates are transverse forces – pressure.
Internal forces which appear in the plate, and induce the deformation, are

- normal forces (tension, compression)
- shear forces (in-plane shear, transverse shear)
- bending moment
- twisting moment



Contents

Simplifications

- for simplification it is often assumed that whole load is carried out by in-plane stresses
- this approach is called membrane theory
- it often works for these part of shells which are far from supporting elements, thickness variations or point loads

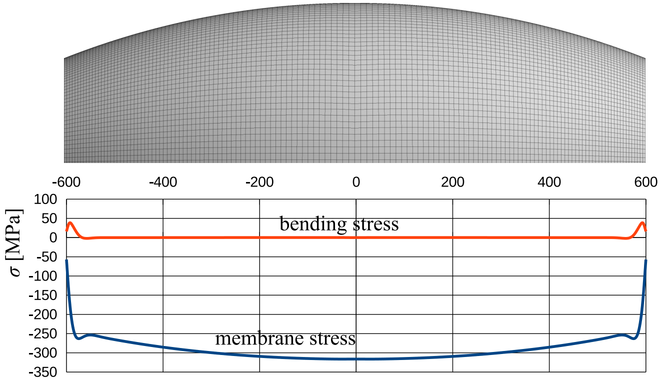
Limitations to the use of membrane theory

For the membrane theory to be applied, the following conditions must be met:

- the edges of the shell are free of transverse shear forces and moments; loads applied to the edges of the shell must lie in a plane tangent to the central surface,
- normal displacements and rotations on the edge of the shell are allowed; this means that the edges are free to move in the direction normal to the central surface,
- the surface of the shell must be smooth and continuous,
- the components loading the surface and edges must be smooth and continuous functions of coordinates.

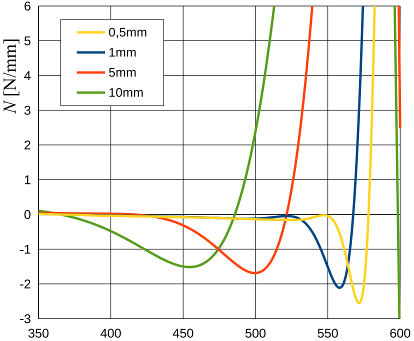
Limitations to the use of membrane theory

- failure to meet any of the above conditions will result in bending stresses in the shell



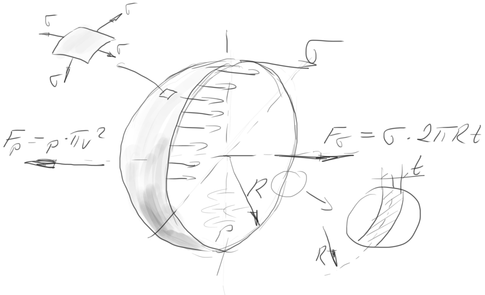
Limitations to the use of membrane theory

- when designing the tank, one should avoid the appearance of bending stresses or strive to minimize them
- bending stresses decrease rapidly as you move away from their source; the thinner the shell, the faster the decay is



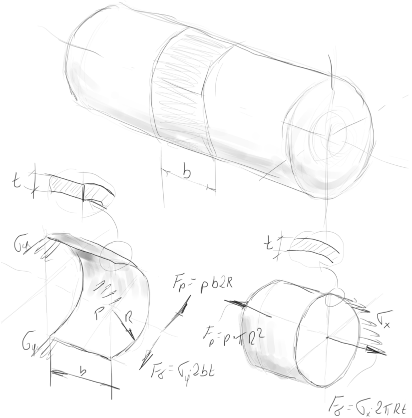
Simple cases
Spherical shell

- let's consider the stress in the spherical tank of the radius R and thickness t



Simple cases
Cylindrical shell

- let's consider the stress in the cylindrical tank of the radius R and thickness t, closed with two dished-heads



Simple cases

Using the memebrane theory, calculate the main stress components as well as von Mises stress for cylindrical tank closed with hemispherical dishedheads.
Given parameters:

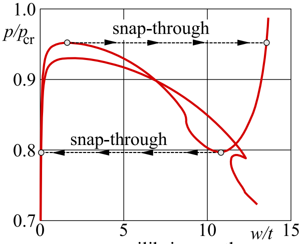
- radius of the tank: $R = 500$ mm,
- thickness of the tank: $t = 5$mm,
- length of the tank: $L = 6000$ mm,
- internal pressure: $p = 1$ MPa.

Contents

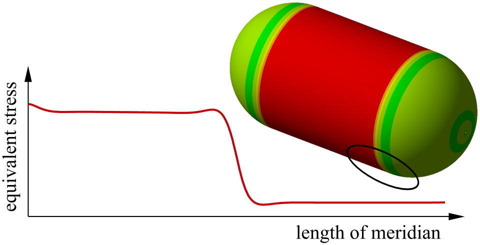
Motivation of the research

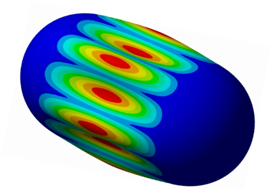
Disadvantages of classical cylindrical tanks

- edge effect
- weak cylindrical part
- unstable equilibrium path



equilibrium path



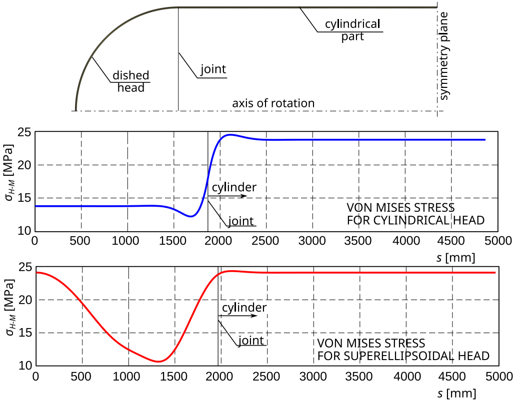


buckling shape

Goal of the research

The goal of the research is to:

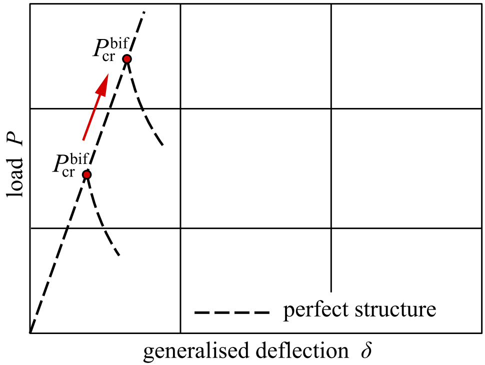
- obtain smooth stress distribution along the whole meridian,



Goal of the research

The goal of the research is to:

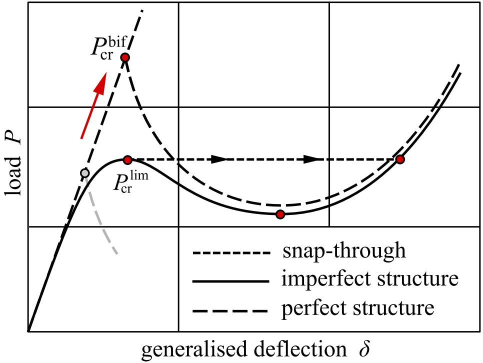
- increase the buckling load,



Goal of the research

The goal of the research is to:

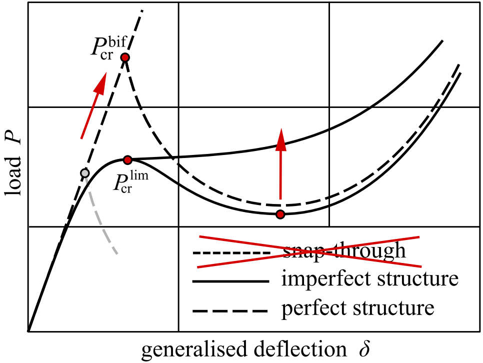
- stabilise the post-buckling behaviour,



Goal of the research

The goal of the research is to:

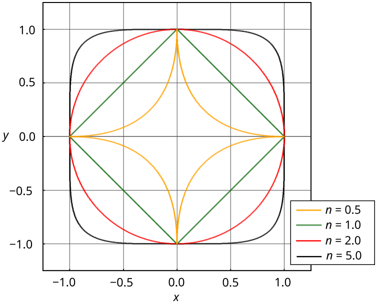
- stabilise the post-buckling behaviour.

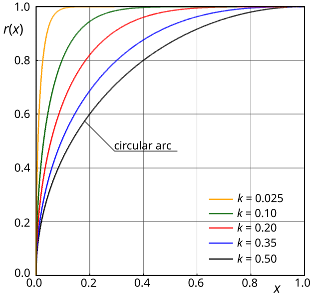


Istniejące rozwiązania

Efekt brzegowy można wyeliminować poprzez:

- zmianę kształtu południka
  - zmiana kształtu dna zbiornika walcowego



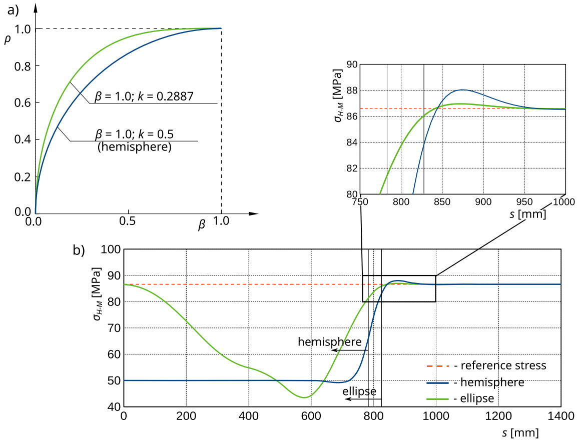


$$\left|\frac{x}{a}\right|^n + \left|\frac{y}{b}\right|^n = 1$$

Istniejące rozwiązania

Efekt brzegowy można wyeliminować poprzez:

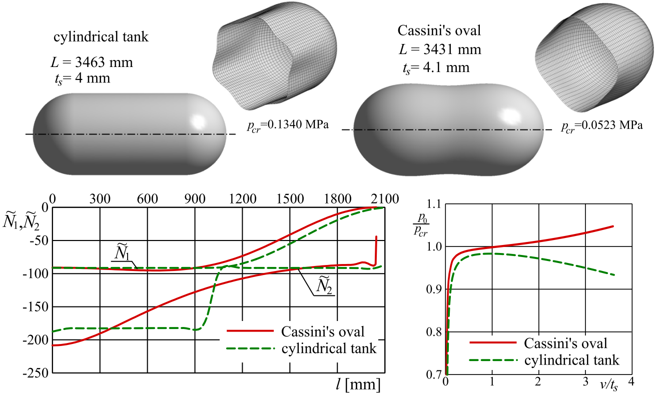
- zmianę kształtu południka
  - zmiana kształtu dna zbiornika walcowego



Existing solutions

The edge effect can be eliminated by:

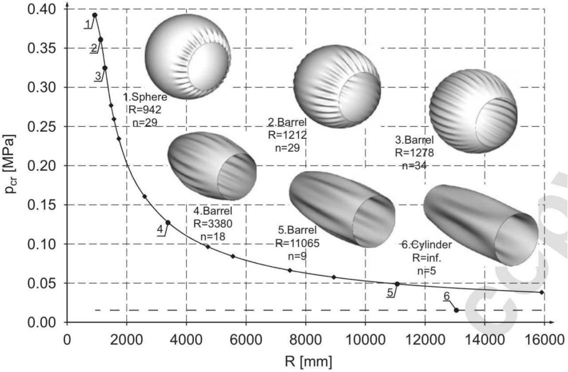
- change of the meridian's shape
  - change the tank's shape



Existing solutions

Critical load can be increased by:

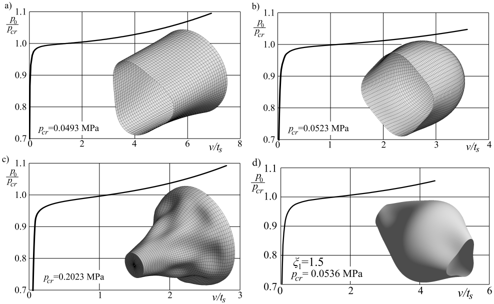
- increase of the shell's thickness,
- introduction of stiffening rings,
- change of the meridian's shape.



Existing solutions

Stabilisation of the post-critical behaviour can be achieved by:

- introduction of stiffening rings
- introduction of initial pretention,
- change of the meridian's shape.

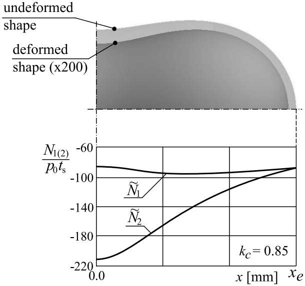


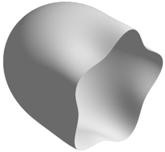
Contents

Complex analysis of shell structure

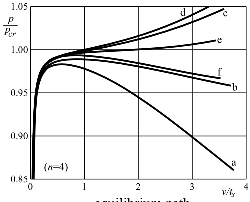
Complex analysis of a shell structure should contains:

- pre-buckling analysis – stress and deformation,
- linear buckling analysis – critical load and critical load,
- post-buckling analysis – equilibrium path.





buckling mode

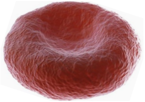


equilibrium path

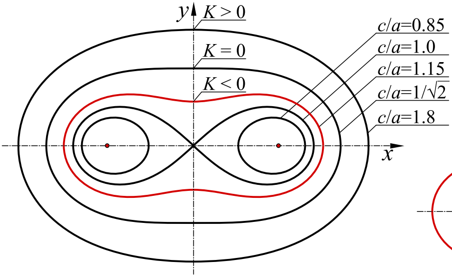
Geometry of Cassini's oval

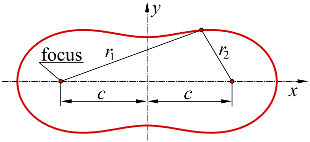
- equation of the curve: $((x - a^2) + y^2)((x + a^2) + y^2) = b^4$
- defined by points: $r_1 \times r_2 = b^2$


cocoon of silkworm


red blood cell


Giovanni Domenico Cassini

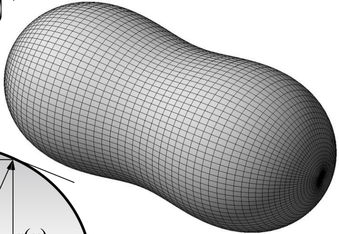


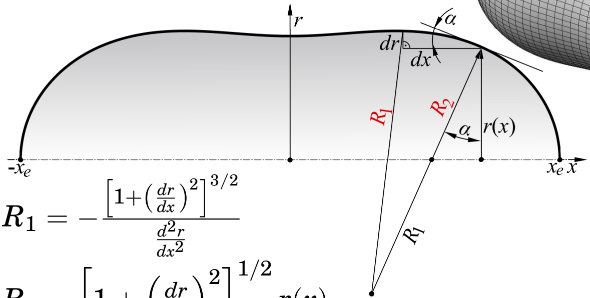


Geometry of Cassini's oval

- equation of the curve:

$$r(x) = \left[\sqrt{4c^2x^2 + a^2} - (c^2 + x^2)\right]^{1/2}$$

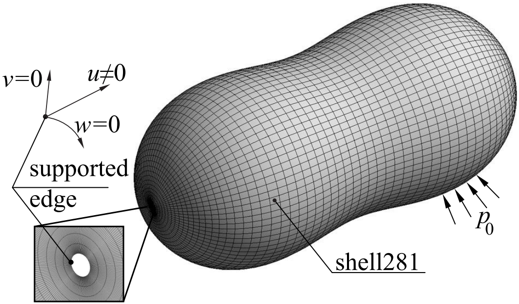




$$R_1 = -\frac{\left[1+\left(\frac{dr}{dx}\right)^2\right]^{3/2}}{\frac{d^2r}{dx^2}}$$

$$R_2 = \left[1 + \left(\frac{dr}{dx}\right)^2\right]^{1/2} r(x)$$

FE model



Assumptions:

- $V = 5\ \mathrm{m}^3$
- $m = 500\ \mathrm{kg}$

Material:

- $E = 200000\ \mathrm{MPa}$
- $\nu = 0.3$

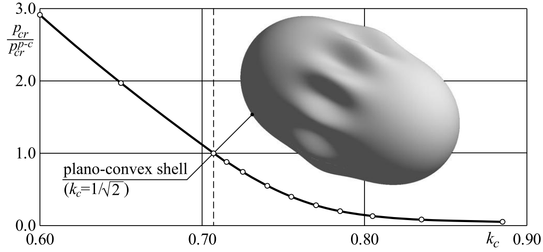
Finite element

- shell element
- 8 nodes
- 6 DOF

The results of Cassini oval analysis

Influence of the curvature of the mid-length of the shell on the value of the critical load

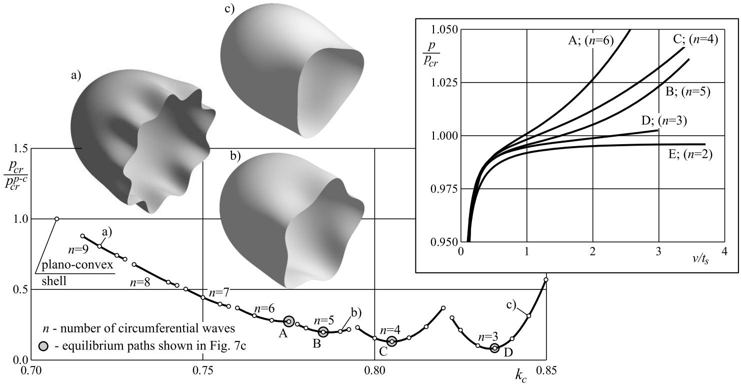
- increasing of the positive curvature increases of the critical load
- decreasing of the negative curvature decreases of the critical load



The results of Cassini oval analysis

Equilibrium paths for selected shells

- for shells of a certain curvature, equilibrium paths starts to stabilise



Literatura

1. Ventsel E., Krauthammer T. *Thin plates and shells*, CRC Press, Boca Raton, 2001
2. Calladine C.R. *Theory of shell structures*, Cambridge University Press, Cambridge, 2007
3. Flügge W. *Stresses in shells*, Springer, 1973