Strength of Mechanical Constructions

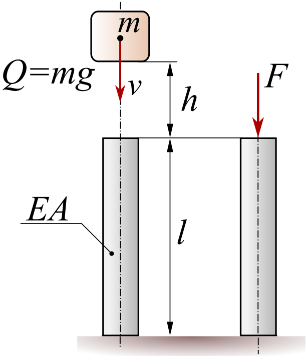
Energy methods in mechanics
Impact load

Paweł JASION, PhD. Eng.
e-mail: pawel.jasion@put.poznan.pl
www: pawel.jasion@pracownik.put.poznan.pl

Poznan University of Technology
Institute of Applied Mechanics
Division of Strength of Materials and Structures

Definition of impact load

- for static load deformation and stress can be easily be determined since the value of force is constant and known in advance
- for dynamic load the force depends on time

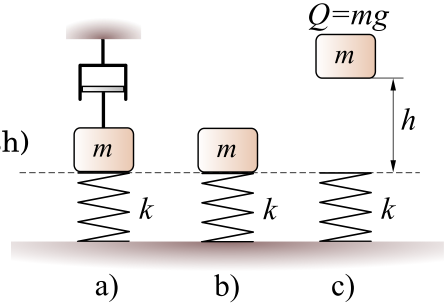


Definition of impact load

Impact load is also called shock, sudden, or impulse load.

Impact load can be divided into three categories depending on the severity

a) rapidly moving force of constant magnitude (car moving on the bridge)
b) suddenly applied load (explosion)
c) direct impact load (crash)



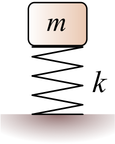
Definition of impact load

Distinction between static and impact load:

- compare the time t required to apply the load with natural period of vibration T of undamped mass on the spring

$$T = 2\pi\sqrt{m/k}$$

  - for $t > 3T$ load is static
  - for $t < 0.5T$ load is dynamic



It should be noted that:

- statically loaded elements are designed to carry the load
- elements subjected to impact load are designed to absorb energy

Determining deformation and stress due to impact

Energy transformation

Deformation and stress due to impact load can be determined using the principle of conservation of energy.

potential energy of elevated mass

$$E_p = mgh$$

$$\downarrow$$

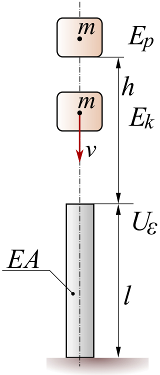
kinetic energy of falling mass

$$E_k = \frac{mv^2}{2} \text{ where } v = \sqrt{2gh}$$

$$\downarrow$$

energy of elastic deformation

$$U_\varepsilon = \frac{F^2 l}{2EA}$$



Determining deformation and stress due to impact

During the impact the kinetic energy is transformed into many different types of energies:

- strain energy of elastic deformation
- production of heat
- local plastic deformation
- kinetic energy of moving further (downward or upward)

Determining deformation and stress due to impact

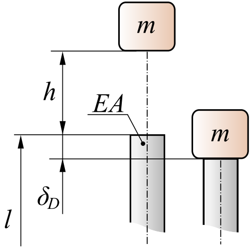
To simplify the analysis the following assumptions are made:

- the mass, after the impact, follows the hit element
- there is no loses of energy
  - all kinetic energy is transformed into elastic strain energy of the hit element; the stress then will be overestimated
- the mass of the hit element is ignored
- stress remain in a linear elastic range
- stress distribution throughout the volume is assumed to be uniform

Determining deformation and stress due to impact

- let's consider a straight bar of the stiffness EA and length l loaded with the mass m falling from the height h.
- the principle of conservation of energy states:

$$E_p = U_\varepsilon$$



  - potential energy is

    $$E_p = mgh = Q(h + \delta_D)$$

  - energy of elastic deformation is

    $$U_\varepsilon = \frac{1}{2} F_e \delta_D$$

- F_e is an unknown equivalent force evoking deformation due to impact.

Impact factor

- comparing the two energies we'll get

$$Q(h + \delta_D) = \frac{1}{2} F_e \delta_D$$

- for a bar under compression, the relation between the force and the displacement has the form

$$\delta_D = \frac{F_e l}{EA} \qquad \text{which gives} \qquad F_e = \frac{\delta_D EA}{l}$$

- by substituting into the above equation and after ordering we have

$$-\delta_D^2 + 2\delta_D \left(\frac{Ql}{EA}\right) + 2h\left(\frac{Ql}{EA}\right) = 0$$

Impact factor

- it should be noted that the expression in parentheses is a deformation under static load Q

$$\delta_S = \frac{Ql}{EA}$$

- thus we have

$$-\delta_D^2 + 2\delta_D\delta_S + 2h\delta_S = 0$$

- the solution of the equation will give the displacement due to impact load

$$\delta_D = \delta_S\left[1 + \sqrt{1 + \frac{2h}{\delta_S}}\right] = \delta_S K_D$$

where K_D is an impact factor

Impact factor

- let's try to determine the equivalent force F_e; we assume that the stiffness of the bar k do not change and equals

$$k = \frac{EA}{l}$$

- since $F_e = \delta_D k$ and $Q = \delta_S k$, we can write

$$F_e = Q\frac{\delta_D}{\delta_S}$$

- thus the equivalent force is equal to

$$F_e = Q\left[1 + \sqrt{1 + \frac{2h}{\delta_S}}\right] = QK_D$$

Impact factor

- in most engineering problems $h >> \delta_S$, thus the impact factor reduces to

$$K_D = \sqrt{\frac{2h}{\delta_S}}$$

- since the impact load is not always the result of gravitation force it is convenient to express the impact factor as a function of kinetic energy

$$K_D = \sqrt{\frac{2h}{\delta_S}} = \sqrt{\frac{2Qh}{Q\delta_S}} = \sqrt{\frac{Qh}{\frac{1}{2}Q\delta_S}} = \sqrt{\frac{E_k^0}{U_\varepsilon^S}}$$

where E_k^0 is the energy of the falling body at the moment of impact

Impact factor

Special case – suddenly applied load

- a special case of the impact load problem is a suddenly applied load for which $h = 0$
- from the formula for dynamic displacement we have

$$\delta_D = \delta_S \left[1 + \sqrt{1 + \frac{2h}{\delta_S}} \right]$$

- and finally

$$\delta_D = 2\delta_S$$

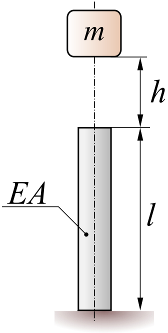
- the impact factor $K_D = 2$; then the suddenly applied load is twice this applied in a static way

$$F_e = 2Q$$

Stresses in bar under impact load

Example 1

Define the dynamic stress σ_D in the bar under compression presented in the figure.



- from the above considerations we know that

$$\delta_D = \delta_S K_D \quad \text{and} \quad F_D = F_S K_D$$

- since according to Hooke's law the stress is proportional to deformation we can write

$$\sigma_D = \sigma_S K_D$$

Stresses in bar under impact load

Example 1

- we will start with determining the impact factor K_D from the relation

$$K_D = \sqrt{\frac{E_k^0}{U_\varepsilon^S}}$$

- energy of elastic deformation for static load is

$$U_\varepsilon^S = \frac{Q^2 l}{2EA}$$

- substituting this to the expression for K_D we have

$$K_D = \sqrt{\frac{2EAE_k^0}{Q^2 l}}$$

Stresses in bar under impact load

Example 1

- multiplying the nominator and denominator by A and knowing that $\sigma_S = Q/A$ there is

$$K_D = \sqrt{\frac{2EA^2E_k^0}{Q^2lA}} \rightarrow K_D = \frac{1}{\sigma_S}\sqrt{\frac{2EE_k^0}{lA}}$$

- and because $\sigma_D = \sigma_S K_D$ finally we obtain

$$\sigma_D = \sqrt{\frac{2EE_k^0}{Al}}$$

Stresses in bar under impact load

- the last expression allows to compare the effect of static and impact load
- under static load stress depends only on the value of the force and the cross-section of the bar
- under impact load stress depends on the volume of the bar (Al) and on the material (E) of the bar
- the same stress will be obtain for a short bar with big cross-section and for a long bar with a small cross-section

Stresses in bar under impact load

- it comes from the character of the loading force
- under static load the force Q is transmitted along the bar and its value do not depends neither on the material nor the dimensions of the bar
- under impact load the loading force is the force F_e and it depends upon the acceleration with which the body suffering impact resists the impacting body i.e. F_e depends upon the time during which the velocity of the impacting body changes

Stresses in bar under impact load

- this time depends on the deformation δ_D that is on pliability of the bar – i.e. the smaller the modulus of elasticity E and the greater the bar length L, the longer is the duration of impact and the smaller are acceleration
- for this reason for damping the impact springs are used which thanks to large deflection increase the time of impact

Stresses in bar under impact load

- the strength condition for impact load can be written as

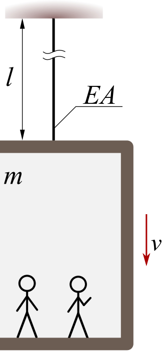
$$\sigma_D \leq \sigma_D^{allow} = \frac{\sigma_Y}{n_D}$$

- the safety factor n_D can be assumed equal to this for static load since the dynamic character of the load has been already taken into account when σ_D was derived
- since the above formulae are approximate n_D should be at least slightly bigger than 2

Structure under impact load

Example 2

The elevator is descending at a constant velocity $v = 2$ m/s. It has the mass of $m = 5$ t and is supported by a single rope of the cross-sectional area $A = 1600$ mm^2. The material of the rope is a steel with $E = 210000$ MPa. Due to some breakdown the elevator stops suddenly being below the top end of the rope about $l = 20$ m.
Determine the maximum elongation and the maximum tensile stress developed in the rope.



Structure under impact load

Example 2

- the maximum value of elongation and stress can be obtained from relations

$$\delta_D = \delta_S K_D \quad \text{and} \quad \sigma_D = \sigma_S K_D$$

- static values can be determined immediately

$$\delta_S = \frac{Ql}{EA} = \frac{mgl}{EA} = 2.92 \text{ mm}$$

$$\sigma_S = \frac{Q}{A} = \frac{mg}{A} = 30.7 \text{ MPa}$$

- the only quantity to be determine is the impact factor K_D

Structure under impact load

Example 2

- the formula for impact factor is

$$K_D = \sqrt{\frac{2h}{\delta_S}}$$

- it is true for the case of free falling weight; in our case the elevator descending at a constant velocity v
- let's transform the above formula having in mind that $v = \sqrt{2gh}$
- solving the above equation with respect to h and substituting this to the formula for K_D we get

$$K_D = \sqrt{\frac{v^2}{g\delta_S}} = 11.8$$

Structure under impact load

Example 2

- thus we have

$$\delta_D = \delta_S K_D = 2.92\ \text{mm} \cdot 11.8 = 34\ \text{mm}$$

$$\sigma_D = \sigma_S K_D = 30.7\ \text{MPa} \cdot 11.8 = 362\ \text{MPa}$$

- from the results it is seen how big can be the difference between the stress resulted from static and impact load
- in the case of static load the material may stay in an elastic range and the same load applied suddenly may cause the plastification of the material

Stresses in bar under impact load

Impact energy capacity

- based on the expression for K_D we can define the impact energy capacity u

$$u = \frac{U_\varepsilon}{V}$$

- in the case of the bar the volume equals $V = Al$ and the energy U_ε equals the energy of kinetic energy at the moment of impact E_k^0
- solving the relation for σ_D with respect of energy we'll obtain

$$u = \frac{\sigma_D^2}{2E} \qquad \left(U_\varepsilon = \frac{\sigma_D^2 V}{2E} \right)$$

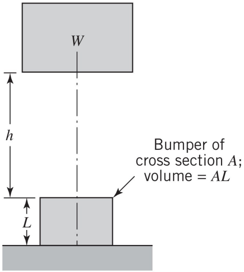
- the above formula allows to compare different damping elements made of different materials

Stresses in bar under impact load

Example 3

A falling weight impacts on a block of material serving as a bumper. Estimate the relative elastic energy absorption capacity of the following bumper materials:

- soft steel: $E = 207000$ MPa; $S_e = 207$ MPa,
- hard steel: $E = 207000$ MPa; $S_e = 828$ MPa,
- rubber: $E = 1.04$ MPa; $S_e = 2.07$ MPa.



Stresses in bar under impact load

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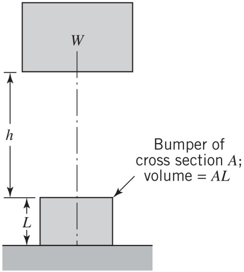
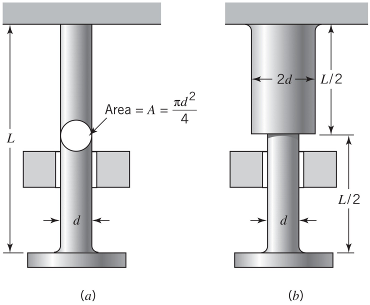


- soft steel: $u = 0.1035$ MPa,
- hard steel: $u = 1.656$ MPa,
- rubber: $u = 2.06$ MPa.

Stresses in bar under impact load

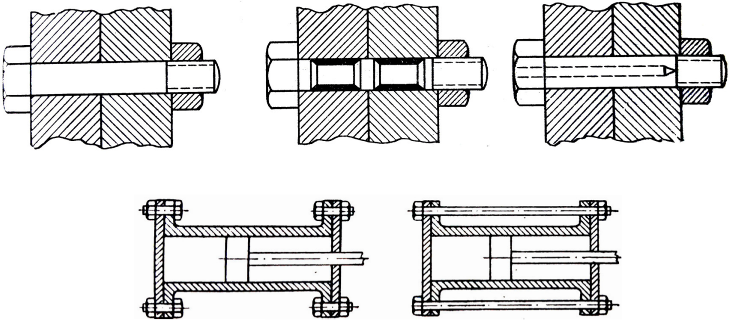
Example 4

Compare energy absorbing capacity of two bars shown on the drawing.
Neglect the stress concentration and assume the elastic limit equal to yield strength S_y.



Stresses in bar under impact load

Examples of design

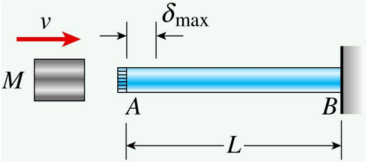


Stresses in bar under impact load

Example 5

A horizontal bar AB of length L is struck at its free end by a heavy block of mass M moving horizontally with a velocity v:

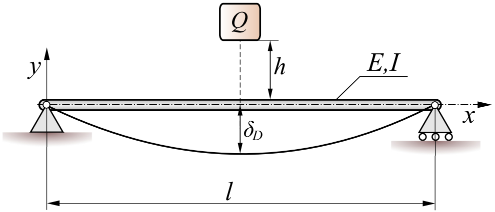
- Determine the maximum shortening δ_{max} of the bar due to the impact and determine the corresponding impact factor,
- Determine the maximum compressive stress σ_{max} and the corresponding impact factor.



Stresses in bar under impact load

Example 5

Determin the maximum value of dynamic displacement δ_D and dynamic stress σ_D.



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